EXCELLENCE QUESTIONS 1

Excellence questions may involve:

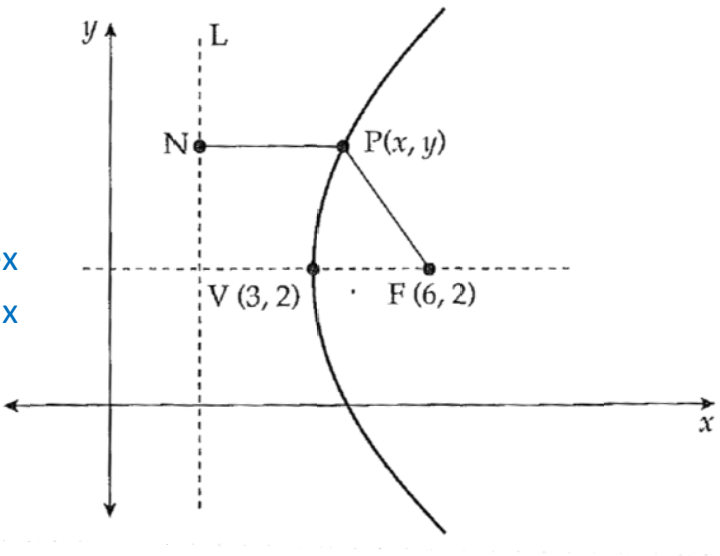
- Knowledge of focus/foci, eccentricity, or directrix
- Proofs
- A chain of reasoning
- Loci

Example 1: *Knowledge of the geometry of conic sections*

Any parabola can be defined as the locus of a point which moves so that it is equidistant from a fixed point (the focus), and a fixed line (the directrix).

A given parabola has its vertex at and its focus at . Let be a point on the parabola and the line L be the directrix.

Write expressions for the lengths PN and PF. Use these to show that the equation of the parabola is .



The directrix is the same distance from the vertex as the focus point is. In this example, the directrix happens to be the y-axis.

The length of PN = .

The length of PF = .

Since PN = PF:

EXCELLENCE QUESTIONS 2

Example 2: *Proof*

A circle is defined by the equation $x^2 + y^2 + 2ax + 2by + c = 0$. Show that the radius of this circle is $\sqrt{a^2 + b^2 - c}$.

Completing the square for x terms and y terms:

$x^2 + 2ax + y^2 + 2by = -c$

$\left[x^2 + 2ax + (\frac{2a}{2})^2\right] + \left[y^2 + 2by + (\frac{2b}{2})^2\right] = (\frac{2a}{2})^2 + (\frac{2b}{2})^2 - c$

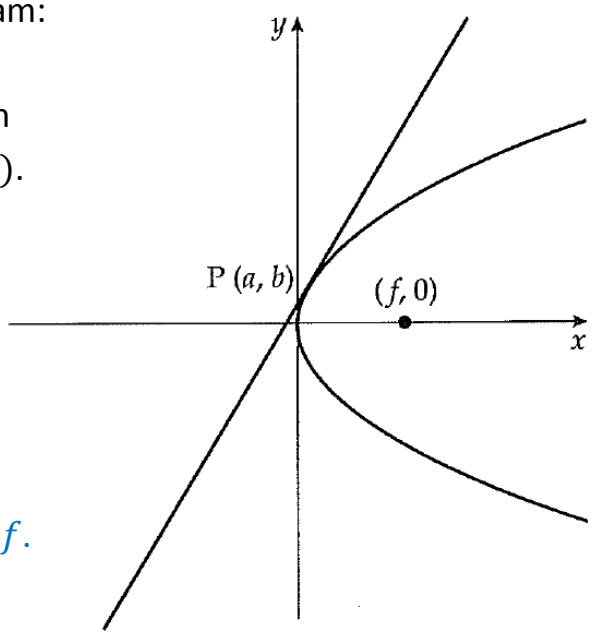
$[x^2 + 2ax + a^2] + [y^2 + 2by + b^2] = a^2 + b^2 - c$

$(x + a)^2 + (y + b)^2 = a^2 + b^2 - c$

So centre of circle is $(-a, -b)$ and radius is square root of the RHS terms: $\sqrt{a^2 + b^2 - c}$.

Example 3: *Chain of reasoning*

At the Pearl Insurance Building in Singapore, there is a steel-latticed radio mast. The radio receivers are parabolic in shape, as show in the diagram:



At the point P (a, b) there is a steel support rod which lies at a tangent to the parabola with focal point $(f, 0)$.

Write the equation of the support rod in terms of a, b, and f.

The general equation for a parabola is $y^2 = 4ax$, where a is the distance of the focus point from the vertex. In this example, the value of a is the constant f. So the equation for this parabola is $y^2 = 4fx$.

Differentiate to get the gradient of the tangent: $2y \cdot \frac{dy}{dx} = 4f$ => $\frac{dy}{dx} = \frac{4f}{2y} = \frac{2f}{y}$

The gradient at the point (a, b) is: $\frac{2f}{b}$

Equation of tangent at point (a, b) is: $y - b = \frac{2f}{b}(x - a)$

$y - b = \frac{2f}{b}x - \frac{2f}{b} \cdot a$ multiply both sides by b

$by - b^2 = 2fx - 2fa$

$2fx - by + (b^2 - 2fa) = 0$ or $y = \frac{2f}{b}x - \frac{2f}{b} \cdot a + b$

Worksheet
- Excellence Revision